Atmospheric Drag Cross Section of Spacecraft Solar Arrays

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Theme

ANALYTIC approximations are derived for average drag cross section of large planar spacecraft solar arrays oriented toward the sun. Effective array cross section is averaged over a single circular orbit revolution, and the dependence on inclination and node, as well as on solar declination, is shown. This result is averaged over complete cycles of nodal regression and solar declination, yielding long-term averages which depend on inclination. Numerical values for both high and low inclinations differ considerably from the random tumbing cross section. The present results are useful in preliminary calculations of drag sustenance ΔV and orbit lifetimes.

Contents

The free molecular drag force is $\frac{1}{2}\rho V^2 C_D S$, where ρ denotes atmospheric density, V the spacecraft velocity relative to the atmosphere, C_D the drag coefficient, and S the cross-sectional area perpendicular to the velocity vector. Drag coefficients have been discussed at length by Cook.[1] He concludes that an average value of C_D slightly greater than 2.2 is appropriate for low altitude satellites, and that for a flat plate at low altitude, C_D exhibits only a minor dependence on the molecular angle of incidence. Moe[2] and Boring and Humphris[3] support these conclusions.

For array area large compared to spacecraft body area, array "shadowing" by the body can be neglected; i.e., the arrays will be in the body wake only for small angles of attack, where they wouldn't contribute much to average drag even if unshaded. The instantaneous array drag varies considerably over an orbit revolution. However, because orbit decay calculations mainly consider long-term effects, average drag is the relevant quantity. For low-altitude circular orbits the variation in C_D during a single revolution is small compared to that in ρS. Hence, C_D can be approximated by its average value. On each revolution the spacecraft will encounter densities for all local times from $0{:}00^h$ to $24{:}00^h$. However, the present analysis will approximate $(\rho S)_{ave}$ by $(\rho)_{ave} \times (S)_{ave}$. Density will be assumed to have a constant value equal to its average, and analytic averaging will be performed only on array cross section. Diurnal density amplitude increases with altitude, restricting the present analysis to circular orbits below about 350 naut miles.

During a given revolution, the sun line makes a constant angle β with the positive orbit normal, where $0 \leq \beta \leq \pi$. Use coordinates (ξ, η, ζ), with ζ being the orbit normal as shown in Fig. 1. Choose the ξ axis such that Earth-sun line lies in the $\xi - \zeta$ plane, and let u denote orbital position. The planar array is assumed oriented perpendicular to the sun line at all times. Let A denote array area, and ψ the angle between the array normal and velocity vector. Instantaneous drag cross section S, is

$$S = A|\cos\psi| = A \sin\beta |\sin u| \quad (1)$$

For circular orbits an average over u is equivalent to a time average. Let S^* denote the average of S over a revolution

$$S^* = \frac{1}{2\pi}\int_0^{2\pi} S\, du = \frac{2A}{\pi}\sin\beta \quad (2)$$

Using Fig. 2 it may be shown that

$$\sin\beta = +[(1 - \cos^2 i \sin^2 \delta) - 2\sin i \cos i \sin\delta\cos\delta\sin\theta - \sin^2 i \cos^2\delta \sin^2\theta]^{1/2} \quad (3)$$

where $\theta = \Omega - \alpha$; α and δ denote solar right ascension and declination, respectively. The array cross section S^*, averaged over one orbit, depends on season as well as inclination and ascending node.

Over several weeks β will vary due to nodal regression and apparent solar motion. A time average of $\sin\beta$, over a

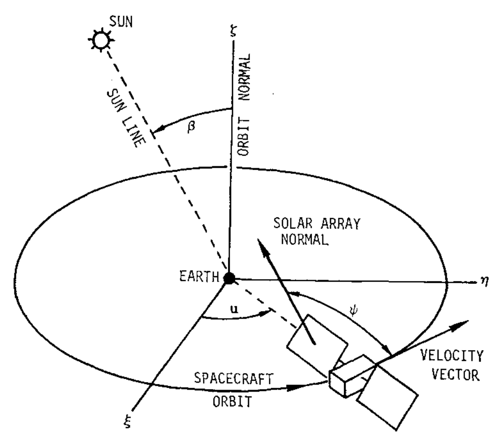

Fig. 1 Solar array orientation relative to spacecraft velocity vector.

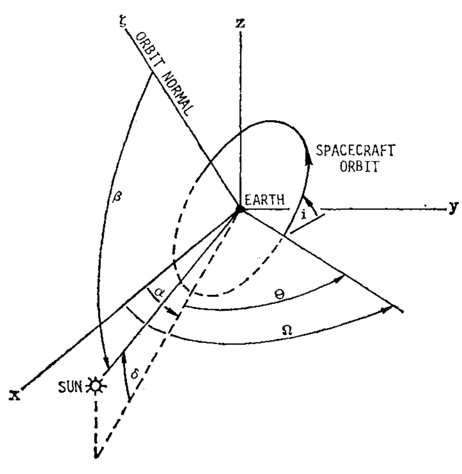

Fig. 2 Relationship of β to other angles.

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complete cycle of δ and/or θ, is required. Variations in mean atmospheric density during the averaging interval, caused by altitude decay or changes in solar activity, will be assumed small. Let P_δ and P_θ denote the cyclic periods of δ and θ, respectively. Three separate cases may be identified: 1) $P_\theta \ll P_\delta$ (low and intermediate inclinations) 2) $\theta \cong$ constant (sun-synchronous orbits) and 3) $P_\theta \approx P_\delta$ (high-inclination orbits).

Low and Intermediate Orbit Inclinations

Ω regresses several degrees per day due to Earth oblateness, whereas α advances 0.986° per day. This suggests averaging over a complete cycle of θ, holding δ constant. An expansion of $\sin\beta$ in powers of $\sin i$ is useful. Because θ varies at an almost constant rate, the time average of $\sin\beta$ over interval P_θ can be approximated by an average over θ from 0 to 2π, denoted by $\langle\sin\beta\rangle$

$$\langle\sin\beta\rangle = \cos\delta + \tfrac{1}{4}\sin^2 i\cos\delta(\tan^2\delta - 1) + \tfrac{1}{64}\sin^4 i\cos\delta(-3 + 6\tan^2\delta + \tan^4\delta) + 0(\sin^5 i) \tag{4}$$

The corresponding cross section $\langle S\rangle$, averaged over one cycle of θ, is $2A\langle\sin\beta\rangle/\pi$. $\langle S\rangle/A$ is plotted vs inclination in Fig. 3 for $\delta = 0°$ and $\delta = \pm 23.5°$. Average cross section depends on the season, and decreases with increasing inclination. The dashed line in Fig. 3 is averaged cross section given by the approximation of random tumbling, which underestimates array drag for low inclinations, and overestimates it for high inclinations, with a maximum error of about 25%.

An average of $\langle\sin\beta\rangle$ over solar declination is useful for estimating orbit lifetimes which exceed 1 or 2 yr, and for calculating drag sustenance ΔV on such orbits. The desired average (with respect to time) can be replaced by an average over α, denoted by $\langle\langle\sin\beta\rangle\rangle$

$$\langle\langle\sin\beta\rangle\rangle = \frac{2}{\pi}E\left\{1 + \frac{1}{4}\sin^2 i\left[\frac{K}{E} - 2\right] + \frac{1}{64}\sin^4 i\left[-9 + 4\frac{K}{E} + \frac{(2-\sin^2\varepsilon)}{\cos^2\varepsilon}\right] + 0(\sin^5 i)\right\} \tag{5}$$

where K and E are complete elliptic integrals of the first and second kinds, respectively, with argument $\sin\varepsilon$(ε is obliquity of the ecliptic, 23.45°). The corresponding cross section $\langle\langle S\rangle\rangle$, averaged over nodal regression and solar declination, is $2A\langle\langle\sin\beta\rangle\rangle/\pi$. $\langle\langle S\rangle\rangle/A$ is plotted vs inclination in Fig. 3.

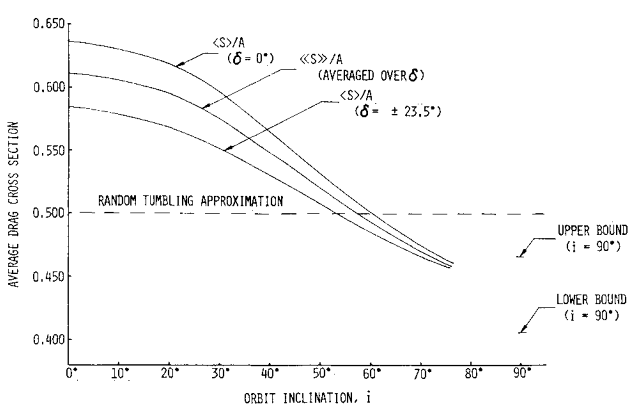

Fig. 3 Average array cross section vs orbit inclination.

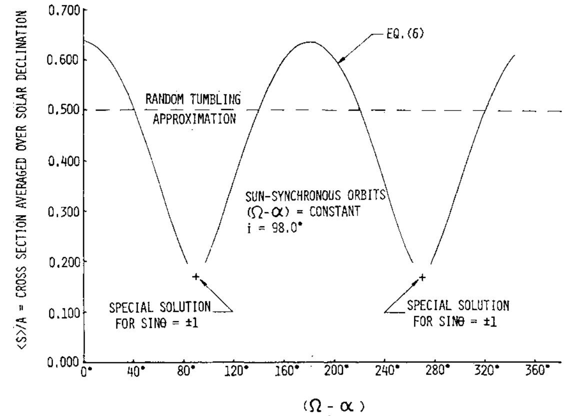

Fig. 4 Average array cross section for sun-synchronous orbits.

For $i = 90°$, both $\langle\sin\beta\rangle$ and $\langle\langle\sin\beta\rangle\rangle$ have the same lower and upper bounds of 0.637 and 0.731, respectively. Plotting these bounds on Fig. 3 indicates that Eqs. (4) and (5) are reasonably accurate even as $i \to 90°$.

Sun-Synchronous Orbits

The angle θ (Fig. 2) has an almost-constant value for sun-synchronous orbits. Below 350 naut miles altitude, inclination must be in the range 96°–98°, so that $|\cos i| < 0.139$. This suggests expanding $\sin\beta$ in powers of $\cos i$, and averaging the result over a complete cycle of δ, holding θ constant. The time average of $\sin\beta$ may be approximated by an average over α, denoted by $\langle\sin\beta\rangle$

$$\langle\sin\beta\rangle = \frac{2}{\pi}(1+N^2)^{1/2}|\cos\theta|\,E(k) + \frac{\cos^2 i\,|\cos\theta|}{\pi(1+N^2)^{1/2}}\left[\left(1 + \tan^2\theta - \frac{1}{\sin^2\theta\cos^2\theta}\right)K(k) + \left(-1 - N^2 + \frac{1}{\sin^2\theta\cos^2\theta}\right)E(k)\right] + 0(\cos^3 i) \tag{6}$$

where

$$N^2 = \sin^2\varepsilon\tan^2\theta, \text{ and } k^2 = \frac{N^2}{1+N^2} \tag{7}$$

Equation (6) is well behaved as $\sin\theta \to 0$. The corresponding cross section $\langle S\rangle$, averaged over the solar declination variation, is $2A\langle\sin\beta\rangle/\pi$, with $\langle\sin\beta\rangle$ given by Eq. (6). $\langle S\rangle/A$ is plotted vs θ in Fig. 4. These results are well suited for estimating long-term orbit sustenance ΔV for sun-synchronous repeating ground track satellites.

References

[1] Cook, G. E., "Satellite Drag Coefficients," *Planetary and Space Science*, Vol. 13, 1965, pp. 929–946.

[2] Moe, K., "Recent Experimental Evidence Bearing on Satellite Drag Coefficients," *AIAA Journal*, Vol. 6, No. 7, July 1968, pp. 1375–1377.

[3] Boring, J. W. and Humphris, R. R., "Drag Coefficients for Free Molecule Flow in the Velocity Range 7–37 km/sec," *AIAA Journal*, Vol. 8, No. 9, Sept. 1970, pp. 1658–1662.